# Oscillations

## Revision and Damping

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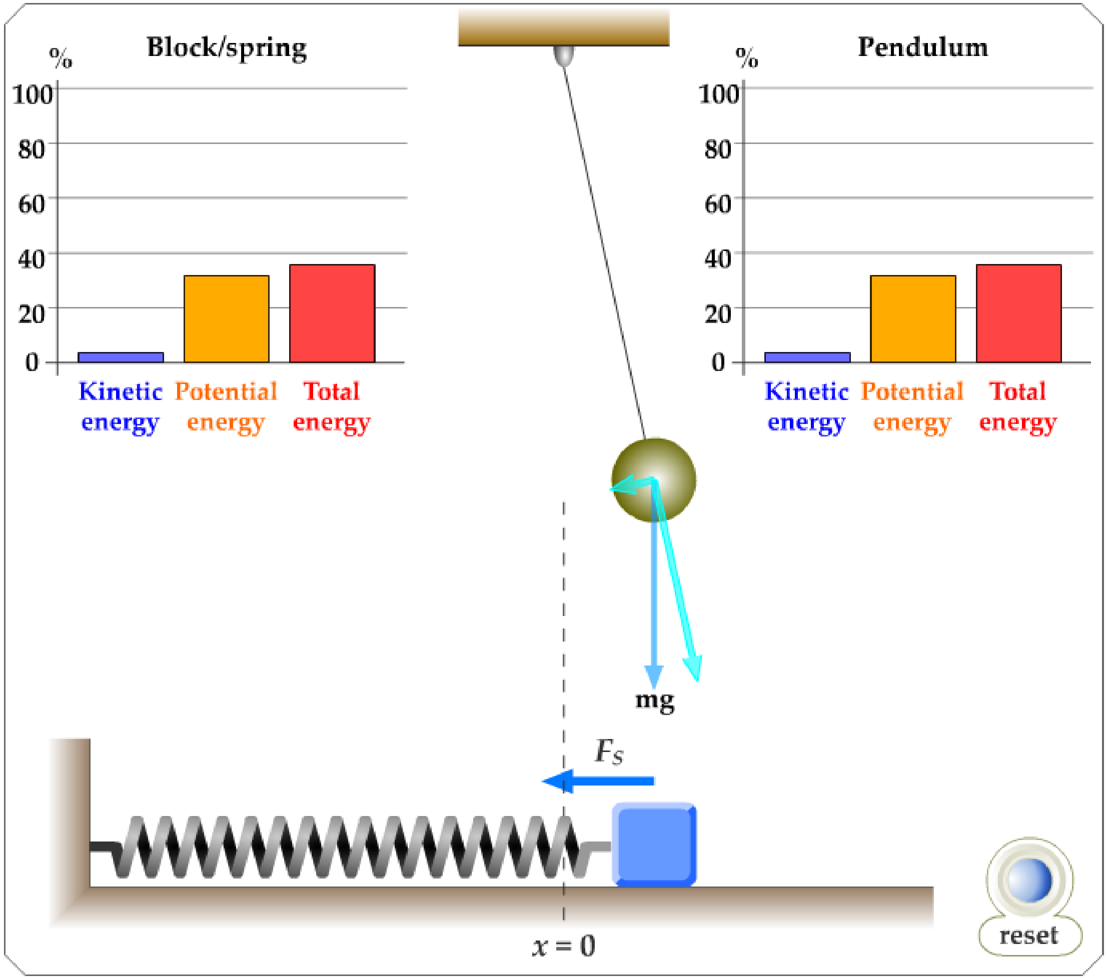
PHYS 202 SS 2013

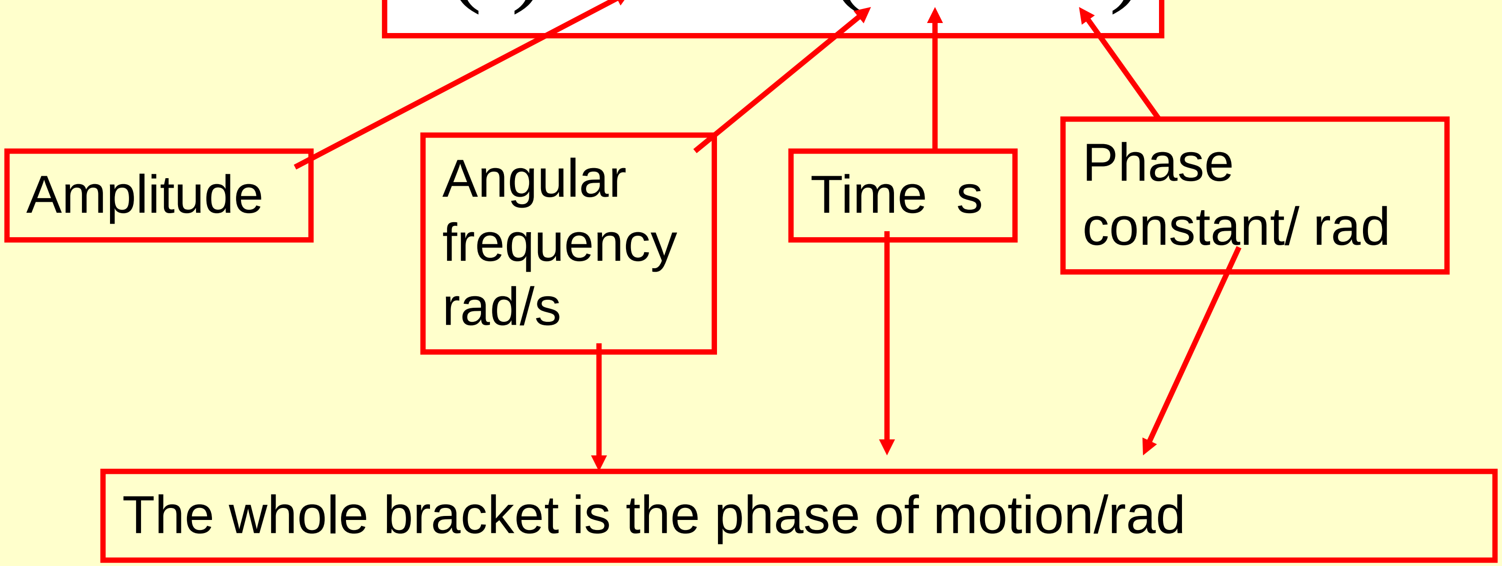
# Simple harmonic oscillation (1510)



# Simple Harmonic motion

- A periodic motion: displacement changes as a periodic function in time,

$$\boxed{x(t) = A\cos(\omega t + \Phi)}$$



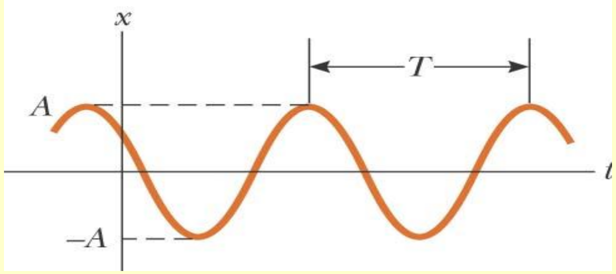
Amplitude

Angular frequency rad/s

Time s

Phase constant/ rad

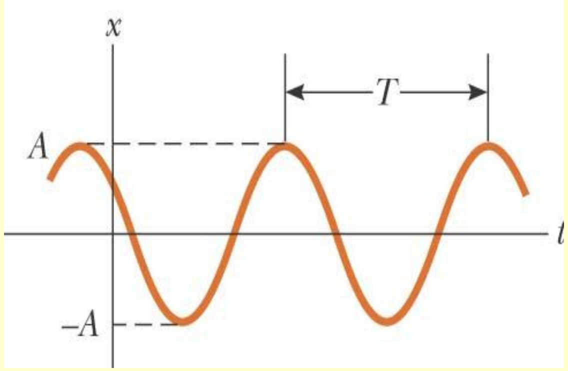
The whole bracket is the phase of motion/rad



# Simple Harmonic Motion

- Characteristics:
- Amplitude A
- Frequency f and periodic time T

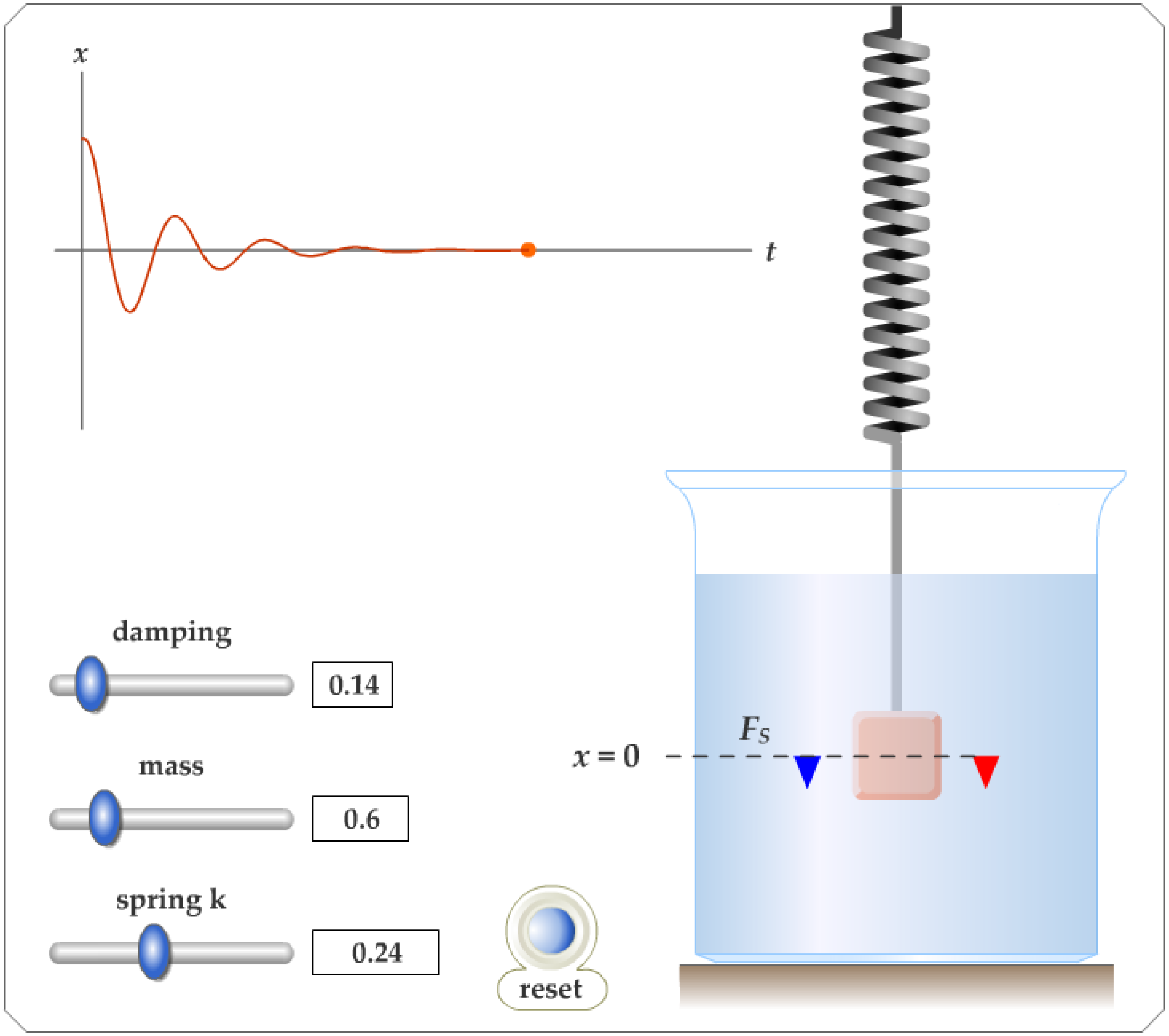
  f=1/T
- Angular frequency  ω = 2πf   (rad/s)
- Phase of the oscillation = (ωt + Φ)
- Phase constant = Φ
- Total energy is constant → conversion of kinetic energy into potential energy and back.



*For a spring mass system*

$$\omega = \sqrt{\frac{k}{m}}$$

# Damped oscillations (1521)



# Damped oscillations

In reality: friction or any resistive force cause damping of the SHM → decrease of the amplitude as time increases.

$$\sum F_x = -kx - b\frac{dx}{dt}. \quad So \qquad -kx - b\frac{dx}{dt} = m\frac{d^2x}{dt^2}$$

Restoring force

Damping Force

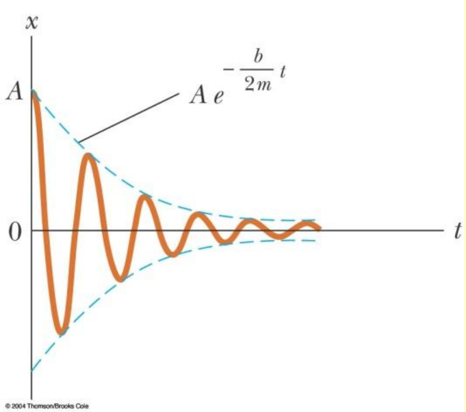
For weak damping („underdamped“):

$$\boxed{x(t) = Ae^{-\frac{b}{2m}t}\cos(\omega t + \varphi)}$$

$$\boxed{\omega = \sqrt{\frac{k}{m} - \left(\frac{b}{2m}\right)^2}}$$

→Amplitude of the oscillation decreases

→Dissipation of Total energy

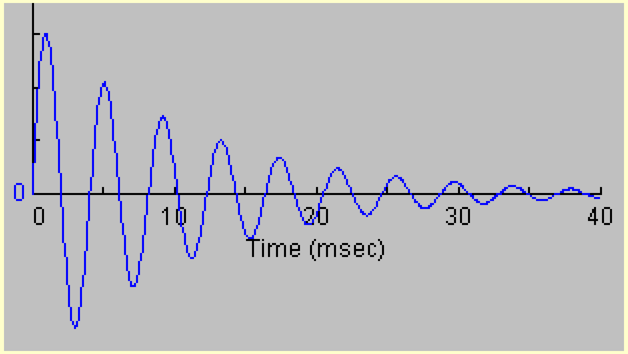


# Damped oscillations

- How strong is the friction?

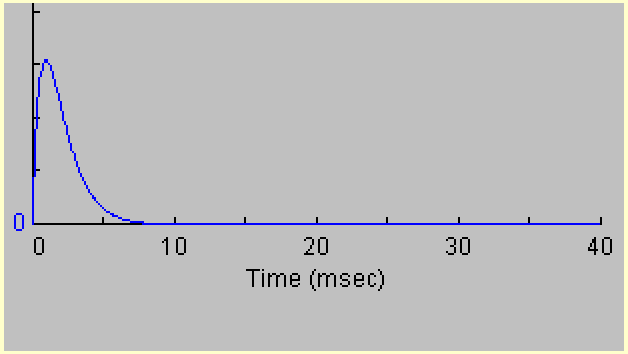
→ underdamped oscillation

$$\omega^2 = \frac{k}{m} - \left(\frac{b}{2m}\right)^2 > 0$$



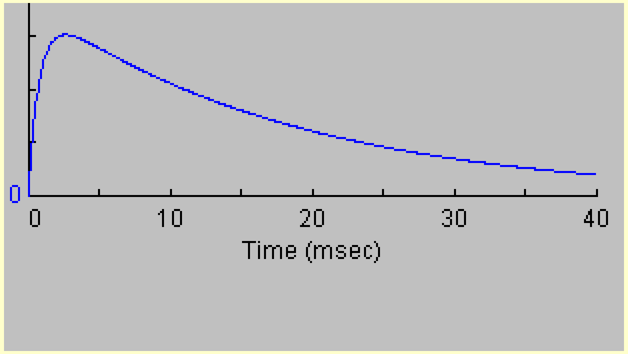
→ critically damped oscillation

$$\omega^2 = \frac{k}{m} - \left(\frac{b}{2m}\right)^2 = 0$$

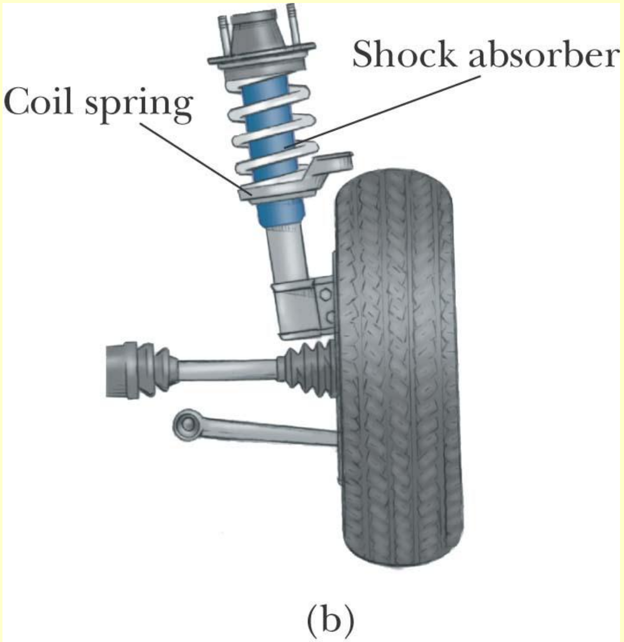


→ overdamped oscillation

$$\omega^2 = \frac{k}{m} - \left(\frac{b}{2m}\right)^2 < 0$$



# For a smooth ride: spring-damper system



(b)

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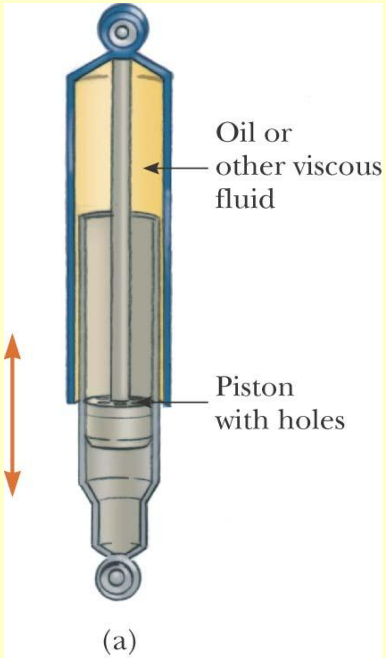

(a)

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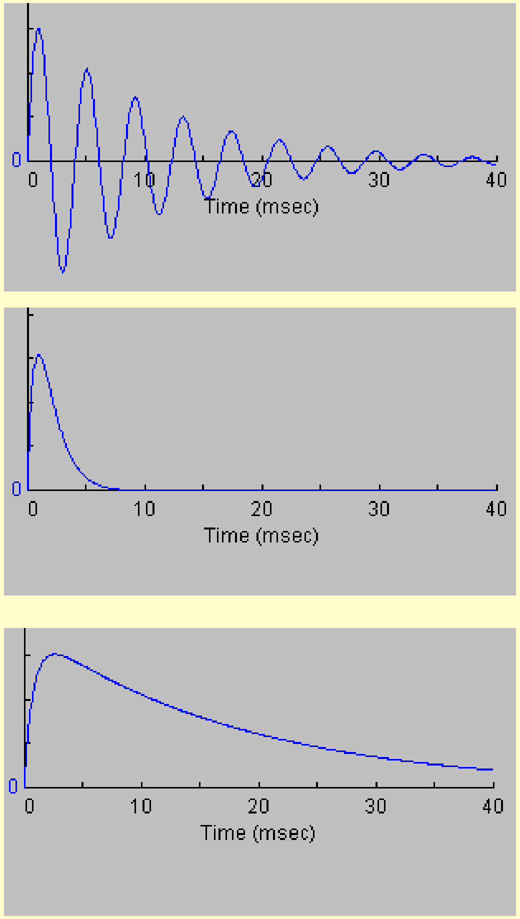


Critical damping would be preferred, Slightly underdamped is accepted for one or two cycles.

# Forced oscillations - Resonance

How to keep an oscillation running despite damping?

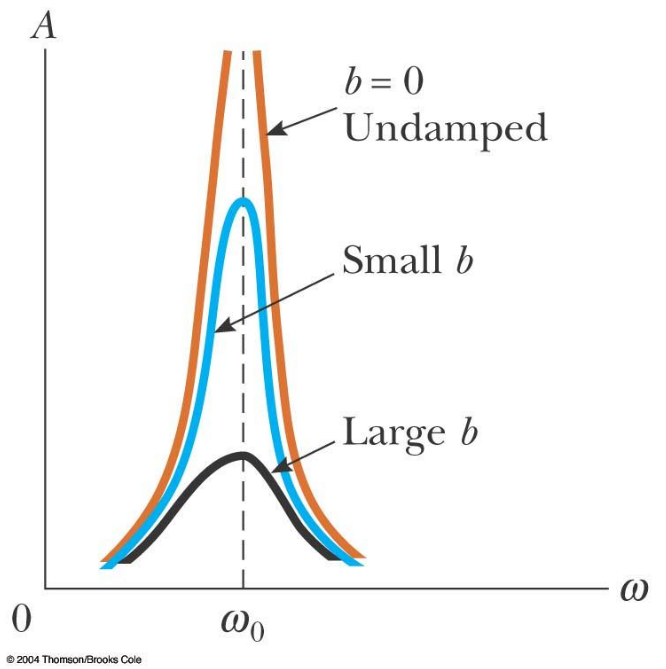
Periodic external force driving the system besides the restoring force and the damping force that provides a substitution for the lost energy.

$$F_{driving} = F_0 \sin(\omega t)$$

System has then
Own frequency: $\omega_0$,
and frequency of the driving force $\omega$

Solution:

$$\boxed{x = A\cos(\omega t + \Phi)}$$



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The system oscillates at the driving frequency, ω. But the Amplitude and phase will depend on $\omega - \omega_0$.

Resonance condition at for $\omega \approx \omega_0$ : max. transfer of energy and the amplitude is max.

**In 1940, turbulent** winds set up torsional vibrations in the Tacoma Narrows Bridge, causing it to oscillate at a frequency near one of the natural frequencies of the bridge structure. (b) Once established, this resonance condition led to the bridge's collapse.